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THERMOCONVECTIVE WAVES IN A HORIZONTAL BOUNDED LAYER OF AN INCOMPRESSIBLE FLUID

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The propagation of finite-amplitude thermoconvective waves in a horizontal fluid layer with rigid boundaries is investigated.

In liquids with a vertical temperature gradient ($\nabla T \uparrow\uparrow g$), as was first shown in [1,2], it is possible for weakly attenuating thermoconvective waves (TCW) to propagate. In [3,4], accurate analytic solutions were obtained for the propagation of small-amplitude TCW excited by temperature oscillations on the vertical wall of a semibounded layer with free edges. The region of weak attenuation of the TCW was determined, the spectral composition of the TCW was investigated, and amplitude and phase characteristics were obtained.

In [5-7], TCW were investigated in fluid layers with rigid boundaries, examining a number of properties of TCW propagation against a background of mechanical equilibrium of the medium and also in conditions of developed natural convection.

In [5, 7], the propagation of periodic temperature perturbations in an air-filled rectangular horizontal cavity (150 × 50 × 11.7 mm) uniformly heated from below was studied experimentally in the frequency range $\omega = 10^{-2}$-10^{-4} sec^{-1}. The amplitude of the temperature oscillations on the side wall did not exceed 10% of the vertical temperature drop. In [6], numerical calculations were carried out for a region of higher frequencies, approximately an order of magnitude larger than the upper limit achieved in the experiment; in this case, the amplitude of the exciting oscillations chosen was half the temperature drop over the height of the layer.

In the present work, the investigation of TCW in bounded fluid layers is continued. Methods of mathematical modeling are used to study the effect of the exciting wave amplitude on TCW propagation and to elucidate possible mechanisms of TCW propagation for different relations between the amplitude of the temperature oscillations on the side wall and the vertical temperature drop in the layer.

Physical experiments [5,7] have shown that TCW propagation proceeds against a background of a two-dimensional cyclic convective structure. As a result, it is possible to limit theoretical investigations to a two-dimensional model, considering TCW in a rectangular region corresponding to a vertical cross section of the layer perpendicular to the axes of the convective cycle.

Mathematical expressions for TCW propagation may be written using the Boussinesq equations [8]. This system of equations contains the Prandtl (Pr), Grashof (Gr), and Rayleigh (Ra) numbers, as well as ω, the frequency of the temperature oscillations on the side wall, and the parameter α characterizing the relation between the amplitude of these oscillations and the vertical temperature drop in the layer [6]. The Grashof number is determined by the total temperature drop in the layer ($|\gamma d| + A_0$) and the Rayleigh number solely by the temperature drop over the height of the layer (γd).

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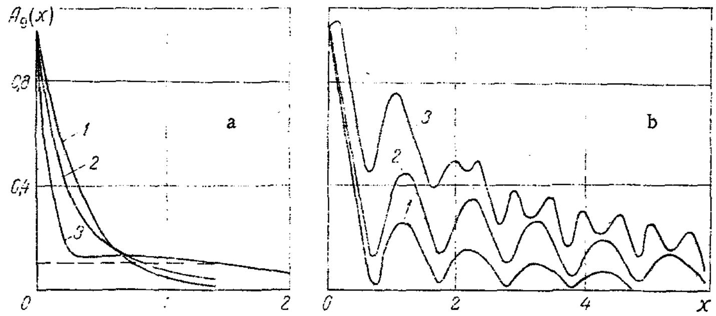

Fig. 1. Amplitude distribution of TCW over median line of layer y = 0.5 for: a) Ra = 0, Gr = 2500 (1), 10^4 (2), 10^5 (3); b) Ra = 1700, α = 2/3 (1); Ra = 2000, α = 2/3 (2); Ra = 10^4, α = 0.5 (3).

The problem was solved numerically on a BÉSM-6 computer, using a monotonic, conservative finite-difference scheme of second-order accuracy based on the method of reversals. A uniform space grid with step h = 1/14 was used. The time variable was discretized automatically in the course of the calculation so as to ensure the stability of the calculation process.

The investigation was carried out for a horizontal region in the form of a rectangle (ratio of sides l/d = 10; l is the length and d is the height). The upper and lower boundaries were assumed to be isothermal with temperatures T_1 and T_2, respectively ($T_1 \geq T_2$). On the side wall, the temperature variation was linear. The TCW were generated by periodic oscillations over time of the temperature on the left-hand side wall:

$$\Theta_{x=0} = 0.5\,(1-|\alpha|)\sin \pi y \sin \omega t + \alpha (1-y). \tag{1}$$

Calculations were carried out for oscillations of maximum amplitude $A_0 = T_1 - T_2$, $(T_1 - T_2)/2$, and $(T_1 - T_2)/4$, corresponding to α = 1/2, 2/3, and 4/5. The case $T_1 = T_2$ (α = 0) was also investigated. The dimensionless frequency ω was varied between the limits 0.5 and 10. The range of Rayleigh numbers considered ($0 \leq \mathrm{Ra} \leq 10^5$) covers TCW propagation both against a background of mechanical equilibrium of the medium and in conditions of natural convection.

For the layer of thickness d = 10^{-2} m used in the experiment of [5], this frequency range corresponds to oscillations with period 2-100 sec. Experimentally, TCW with periods 600 sec and more have been studied.

In isothermal conditions ($T_1 = T_2$, α = 0, Ra = 0), temperature perturbations caused by periodic temperature oscillations on one of the side walls of the layer rapidly attenuate, the character of the attenuation being determined by the frequency and amplitude of the temperature oscillations at the wall. The amplitude of these oscillations is characterized by the parameter Gr. In the course of the numerical experiments it was established that, for the investigated frequency range ($0.5 \leq \omega \leq 10$), the effect of Gr on the propagation of the temperature perturbations is significant only when Gr > 2500. For Gr < 2500, the propagation of the temperature perturbations in the fluid layer is virtually identical to that in a solid medium; in other words, we are dealing with ordinary temperature waves. For Gr ≥ 2500, the effect of convection leads to marked distortion of the curves characterizing the change in maximum amplitude of the temperature perturbations along the layer (Fig. 1a). Analysis of the behavior of this curve shows that, close to the side wall on which temperature modulation occurs, the temperature perturbations attenuate more rapidly than in a solid medium, the maximum amplitude of the temperature oscillations $A_\Theta(x)$ at a distance x ~ 0.7d from the side wall being reduced by a factor of about 5. Subsequently, however, the decrease in $A_\Theta(x)$ along the layer is greatly slowed, so that the perturbations penetrate into the layer to a much greater depth than in the case of a solid medium. Increase in A_0 (and hence in Gr) leads to an intensification of the effect. For Gr ≥ 10^5, a weakly expressed maximum on the amplitude curve is even observed.

If the depth of penetration L of the temperature perturbations is defined as the distance from the side wall at which their amplitude is smaller by a factor of 10 than the amplitude at the wall, it may be observed that, with increase in Gr, L increases from 0.7d (for Gr ⩽ 2500) to 1.2d (for Gr = 10^5) and in the considered frequency range does not depend on ω (Fig. 2).

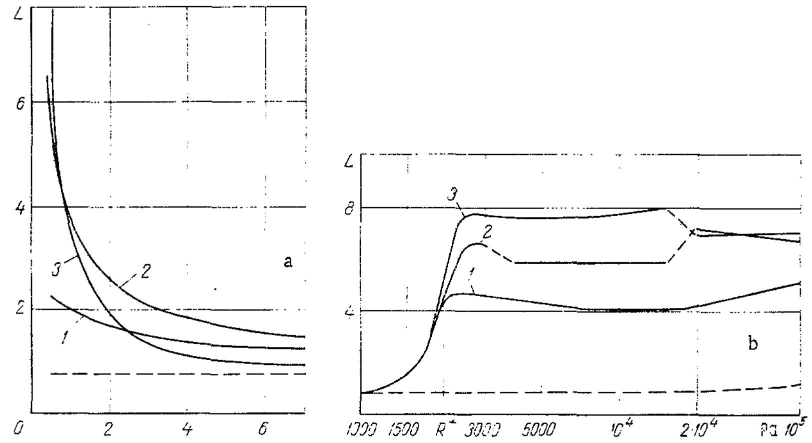

Fig. 2. Dependence of the depth of penetration L of TCW: a) on ω (plotted along the abscissa) for Ra = 1650 (1), 2500 (2), 10^4 (3); b) on Ra for ω = 0.5 and α = 0.8 (1), 0.5 (2), 2/3 (3).

The wavelength λ calculated from the phase shift decreases from 70d to 36d as ω changes from 0.5 to 10 (Gr $\leq$ 2500), which is in good agreement with the results of analysis [4]. Further increase in Gr leads to negligible increase in λ.

This character of the propagation of temperature perturbations of the form of Eq. (1) in an isothermal fluid layer determines the structure and intensity of the convective motion.

In this case, the layer contains a single convective cell, in which the direction of fluid circulation changes periodically, synchronously with, but in opposition to, the temperature oscillations of the side wall. As Gr increases, the intensity of fluid circulation in the cell rises, and the cell itself increases in size. Thus, for example, whereas for Gr = 2000 the convective motion covers the region $0 < x \leq 1.5$, for Gr = 10^5 intensive motion of the fluid is observed in the region $0 < x \leq 3$. The noted property of TCW propagation in an isothermal fluid layer is in good qualitative agreement with the data of [4, 5].

The propagation of temperature perturbations in a horizontal fluid layer with a temperature difference ($T_1 > T_2$) corresponding to Ra $\leq$ 1200 between top and bottom scarcely differs from the isothermal case ($T_1 = T_2$); and, although in this case the layer contains not one but several convective cells, the intensity of fluid circulation in it falls rapidly with increasing distance from the side wall at which temperature modulation is occurring. For Ra = 1000, for example, the intensity of fluid circulation falls by almost an order of magnitude on passing from one cell to the next. The amplitude curve shows a series of alternating maxima and minima, rapidly decreasing with distance from the side wall (Fig. 1b). The maxima on the amplitude curve correspond to the interfaces between cells, where alternation over time of upward and downward flow occurs; the minima correspond to the centers of the cells.

The changes in intensity and direction of the fluid circulation in response to temperature oscillations at the side wall occur practically simultaneously in all the cells. The phase shift in neighboring cells (leaving out of account a phase change of π on passing from cell to cell) is negligible, and corresponds to a wavelength of the traveling wave $\lambda \simeq$ 70d–30d for ω = 0.5–10.

As Ra increases and approaches the critical value Ra*, there is not only increase in the intensity of the convective flow, but also equalization of the fluid circulation velocity between the cells. Accordingly, there is a significant increase in the depth of penetration L of the TCW (Fig. 2b). In addition, decrease in the frequency of temperature modulation at the side wall leads to increase in L (Fig. 2a). Thus, for Ra = 1650, when ω decreases from 10 to 0.5, L increases by a factor of 1.7. Evidently, this property is only significant in regions of comparatively high frequency ($\omega > 0.1$). For $\omega < 0.1$, in the subcritical range of Rayleigh numbers, the frequency of the temperature oscillations at the side wall is found to have no effect on L [5].

Increase in intensity of circulation of the fluid in the convective cell leads to an increase in inertia, and this, in turn, entails a decrease in wavelength (Fig. 3b). As Ra approaches Ra*, the phase shift markedly increases and hence the propagation of the wave front along the layer can be traced up to the moment of its extinction.

In the steady state, when the temperature of the side wall deviates from the equilibrium value (linear profile), a new cell is formed close to it and the cell adjacent to the wall is compressed to almost one-half. In the next moments

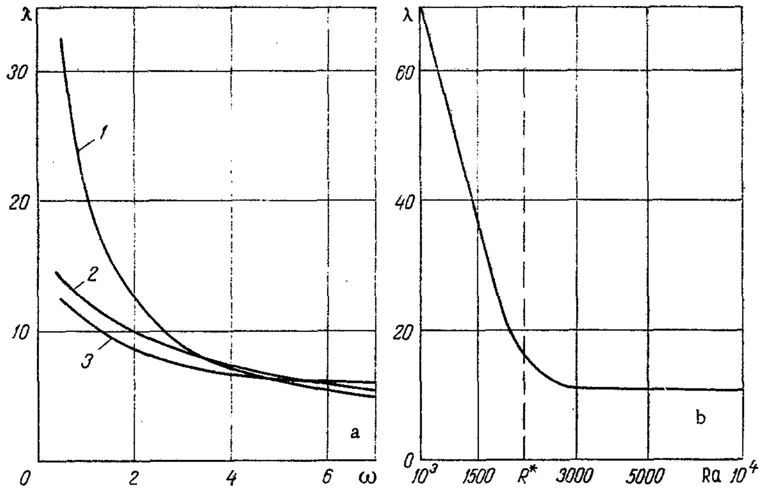

Fig. 3. Dependence of wavelength: a) on ω for $\alpha = 2/3$ and Ra = 1500 (1), 2500 (2), 10^4 (3); b) on Ra for $\omega = 0.5$.

the compressed cell reverts to its normal state and the following cell is compressed, and so on. It may be noted that this picture is also observed in a layer with free boundaries [4]. The distance between two successive compressions is half the wavelength.

For Ra ≥ Ra*, the TCW propagation occurs against a background of developed cell convection. In [5,6] it was noted that in this case the propagation of the temperature perturbations occurs as a result of the periodic appearance and disappearance of an additional cell close to the wall on which the temperature oscillations are occurring. When the additional cell appears, the neighboring cell is compressed and detached from the side wall. Then the additional cell diminishes and disappears, producing a region with normal cells (mode I). The TCW propagates in the form of alternating regions with compressed and normal cells.

However, it is found that in certain conditions other mechanisms of TCW propagation may be realized. For example, if the amplitude of the temperature oscillations at the side wall is insufficient for the formation of an additional cell, the formation of regions with compressed and normal cells occurs as a result of the periodic disappearance and appearance of the cells already present at the wall (mode II). For exciting oscillations of small amplitude, TCW may be generated as a result of the periodic expansion and compression of cells at the side wall. In this case, the number of convective cells in the layer remains constant (mode III).

It should be noted that it is not solely the amplitude of the exciting oscillations (the parameter α) which determines the realization of a particular mode of TCW excitation, but also to a large extent ω and Ra. At frequency $\omega = 0.5$ and $\alpha = 0.5$, for example, mode I is realized for Ra in the range $5 \cdot 10^3 \leqslant \mathrm{Ra} \leqslant 10^4$ but mode II for Ra outside that range ($\mathrm{Ra} < 5 \cdot 10^3$, $\mathrm{Ra} > 10^4$). For $\alpha = 2/3$ ($\omega = 0.5$), mode II is realized for $\mathrm{Ra} < 1.5 \cdot 10^4$ but mode III for $\mathrm{Ra} > 1.5 \cdot 10^4$.

To a large extent, the particular mode determines the depth of penetration L of the TCW, and a change in the mode leads to a sharp change in L (Fig. 2b). Mode II favors the propagation of weakly attenuating TCW, while the depth of penetration of TCW is least for mode III.

For Ra ≥ Ra*, the depth of penetration of TCW depends strongly on the frequency of the initial perturbation ω. With decrease in ω, the value of L rapidly increases (Fig. 2a). Especially rapid growth in L is observed for $\omega < 1$. Experimental results for $\omega < 0.1$ [5] also show an increase in L with decrease in ω, but there is found to be a lower frequency limit ($\omega \sim 10^{-4}$ sec^{-1}) below which decrease in ω produces no change in L [5]; this is evidently associated with the attainment of a quasisteady state.

Investigation shows that, in the subcritical region, increase in Ra is accompanied by an increase in L, which gradually slows and stabilizes when Ra ~ 2500 (Fig. 2b).

Analysis of the curve characterizing the change in maximum amplitude of the temperature oscillations along the layer shows that, right up to Ra ~ 5000, its general form is much the same as in the case Ra < Ra*, although the extremal values of the amplitude are much higher in the subcritical region of Ra (Fig. 1b). For Ra > 7000, the individual maxima on the amplitude curve are split into two components, following the formation of isothermal nuclei at the centers of the convective cells.

Let us consider the dependence of the wavelength λ on the Rayleigh number, and also on the amplitude and frequency of the exciting oscillations. Calculation shows that, in the above-critical region (Ra > Ra*), λ is practically independent of Ra and α (Fig. 3b). Hence the phase velocity of TCW propagation is also independent of Ra and α and is only a function of ω. In fact, increase in ω leads to decrease in λ; the decrease is most significant for small ω ($\omega < 3$) and for large frequencies the dependence of λ on ω markedly decreased (Fig. 3a).

In conclusion, it should be noted that the results on TCW obtained in the present work by numerical calculation are in good qualitative agreement with those of analysis [3, 4] and of physical experiments [5, 6].

NOTATION

$Gr = \beta g d^3(|\gamma d| + A_0)/\nu^2$, Grashof number; $Ra = Gr\alpha Pr = \beta g d^3 \gamma d/\nu a$, Rayleigh number; $\alpha = \gamma d/(|\gamma d| + A_0)$, parameter characterizing the relation between the vertical temperature drop in the layer and the amplitude of the temperature oscillations at the wall; $\gamma = (T_1 - T_2)/d$, vertical temperature gradient in layer; l, length of layer; d, layer thickness; A_0, maximum amplitude of temperature oscillations at wall; ν, kinematic viscosity; a, thermal conductivity; $Pr = \nu/a$, Prandtl number; β, coefficient of thermal compressibility; g, acceleration due to gravity; $\Theta(x,y,t)$, dimensionless temperature in layer; Ra*, critical Rayleigh number corresponding to loss of mechanical equilibrium of the layer; ν/d, scale of velocity; d^2/ν, scale of time; ω, frequency of exciting oscillations; L, depth of penetration, defined as the distance from the side wall at which the amplitude is reduced by a factor of 10; $A_\Theta(x) = 2(1-|\alpha|)^{-1}[\max\Theta(x;\ 0.5;\ t) - \min\Theta(x;\ 0.5;\ t)]$, amplitude of temperature oscillations in median line of cavity $y = 0.5$, $t \in [t_1, t_2]$, $t_2 - t_1 = 2\pi/\omega$.

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APPROXIMATE SOLUTION OF EXTENDED GRAETZ PROBLEM BY ORTHOGONAL COLLOCATION

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The method of orthogonal collocation is applied to the Graetz problem. The method allows a very accurate solution to be obtained in the initial region, where the Fourier series converges very slowly.

1. Introduction

Linear partial differential equations (LPDE) are the mathematical models most commonly used to describe engineering systems. Boundary-value problems for these equations may be solved by means of Fourier

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